

Introduction

Remember the recipe for the uniform distribution: for equally likely outcomes,

$$P(\text{event}) = \frac{\text{number of outcomes in the event}}{\text{total number of possible outcomes}}.$$

For each problem, three questions worth asking yourself: would a decision tree help? Is this sampling **with** or **without** replacement? Does **order** matter?

Problems

1. A bakery has 7 different pastries.
 - (a) In how many different orders could all 7 be lined up in the display case?
 - (b) Only the 3 front positions are visible from the street. In how many different ways could those 3 visible spots be filled (choosing from the 7 pastries, order along the row mattering)?

2. Four friends – Priya, Quinn, Rosa, and Sam – sit in a row of 4 chairs, with every ordering equally likely.
 - (a) How many possible seating orders are there?
 - (b) What is the probability Priya sits in the first chair?
 - (c) What is the probability Priya is first **and** Quinn is second?

3. Counting problems
 - (a) A breakfast deal lets you pick one drink (coffee or tea) and one pastry (croissant, muffin, or bagel). Draw a decision tree showing every possible breakfast deal, and state how many there are.
 - (b) A 4-digit PIN uses digits 0–9 and digits are allowed to repeat. How many possible PINs are there?

4. Counting problem Part 2

- (a) A class has 12 students. How many different groups of 4 students could be chosen to present together?
- (b) A pizza shop has 8 toppings. How many different 3-topping pizzas can you build (all toppings different, order irrelevant)?
- (c) In one sentence, explain why part (b) is a combination (n choose k) and not a permutation.

One number will be drawn at random from each of the two sets below:

$$A = \{1, 2, 3\}; B = \{1, 2, 3, 4\}$$

5. What are the total number of possible outcomes? Draw a decision tree to list them all.

6. What is the probability the number drawn from A is greater than the one drawn from B ?

7. What is the probability that the number drawn from A is equal to the one drawn from B ?

8. What is the probability the number drawn from A is smaller than the one drawn from B ?

Consider a box which has 4 tickets inside labeled 0,1,2,3. Say we pick tickets at random from a box (so each ticket in the box is equally likely each time you pick). Let A be the event that the *first pick* yields an even number; B be the event that the *second pick* is greater than or equal to one.

9. Pick two numbers without replacement. Find $P(B \mid \text{first pick is } 0)$.

10. Pick two numbers without replacement. Find $P(B \mid \text{first pick is } 2)$.

11. Pick two numbers with replacement. Find $P(B \mid A)$.

12. You roll a fair six-sided die 3 times.

- (a) How many equally likely outcome sequences are there? (Treat the rolls as ordered, e.g. (2, 5, 2).)
- (b) What is the probability all three rolls are 6?
- (c) What is the probability **no** roll is a 6?
- (d) What is the probability of getting **at least one** 6? (Hint: use part (c).)

13. You flip a fair coin 5 times, recording the sequence of heads and tails.

- (a) How many equally likely outcome sequences are there?
- (b) How many of those sequences contain exactly 3 heads?
- (c) What is the probability of getting exactly 3 heads?
- (d) What is the probability of getting no heads at all?

14. In a class of 25 students, 5 will be chosen to be on student council.

a) How many possible student councils are there?

b) Say you are a student in the class. What is the probability you will be chosen to be on the council?

15. Say each day there is a 10 percent chance of rain. What is the probability I will have to wait exactly 10 days until it rains? (It rains on the 10th day but not before).

Coding Questions

See the tutorial section of the notes to see the general code structure for simulating many trials using a for loop. Pick one of the problems earlier in the worksheet with a specific computed probability. Write R code to simulate that event 10000 times and see if you can get a frequency close to the computed probability. See if the simulated proportion is close to the one you computed above.

The general structure of your R code should look like

```
#make state space vector

#CODE HERE

#number of simulations

N <- 10000

#make an empty place holder vector to store outputs

my_holder_vec <- NULL

# make a for loop to run as many times as wanted
for(i in 1:N){
  #simulate one outcome

  #CODE HERE

  #check if it is in the event

  did_event_occur <- #CODE HERE

  #store true or false in my place holder vector
  my_holder_vec[i] <- did_event_occur
```

```
}  
#compute proportion of times event occurred  
mean(my_holder_vec)
```