Consider picking tickets at random from a box (so each ticket in the box is equally likely each time you pick). Suppose the box has *four* tickets marked 0, 1, 2, and 3 respectively.

Let A be the event that the *first pick* yields an even number; B be the event that the *second pick* is greater than or equal to one.

1. Pick two numbers without replacement. Find $\mathrm{P}(B \mid \text{first pick is } 0)$.

2. Pick two numbers without replacement. Find $\mathrm{P}(B \mid \text{first pick is } 2)$.

3. Pick two numbers with replacement. Find $\mathrm{P}(B \mid A)$.

Consider a fair, eight-sided die.

4. I roll the die four times. What is the probability that I roll the **same** number on all four rolls?

5. I roll the die twice. What is the probability that the rolls are **different**?

My dog Bella has two toys that she loves: an orange ball, and a thick rope. Each time she picks out a toy, she chooses it independently of all the other times (like a coin toss). That day, she was busy, so went to her toys only **three** times.

Define the events A and B where:

A is the event that she picked the rope *at most* one time;

B is the event that the toys she picked that day included *both* the rope and the ball.

6. Are A and B independent?

An American roulette wheel has 38 pockets, of which 18 are red, 18 black, and 2 are green. In each round, the wheel is spun and a white ball lands in one of these 38 pockets.

7. What is the probability of getting at the ball landing in a green pocket *at least once* in 5 spins of the wheel?

A European roulette wheel has 37 pockets, of which 18 are red, 18 black, *and only 1 green.* The roulette wheel is numbered 0 through 36.

8. Write R code to simulate three spins of this wheel.

9. Now imagine that after each of the three spins, a pocket disappears. Simulate three spins of this magic wheel.

We will now perform our first simulation of the year! For the following questions, consider the **European** roulette wheel defined above and ensure your Quarto document will present the same results each time it is rendered. Write your code in the spaces below.

10. Create three vectors: one which contains 100 simulated spins of the European roulette wheel (call this `one_hundred`), one which contains 1,000 such spins (call this `one_thousand`), and another which contains 10,000 such spins (call this `ten_thousand`).

11. Create a new vector that returns `TRUE/FALSE` values for each element in `one_hundred`, where `TRUE` means that the number spun is greater than 18, and save it. Repeat these steps for the `one_thousand` and `ten_thousand` vectors.

12. Find the proportion of numbers spun in each simulation that were greater than 18 (write the code and the proportion). *Hint: how can you take a proportion of a logical vector?*

13. Comment on how the proportions changed with respect to the true probability of spinning a number greater than 18 as the number of spins increased.

Suppose A and B are non-empty events such that $P(A) = 0.5$ and $P(B) = 0.7$.

14. Is it possible for A and B to be mutually exclusive? Why/ Why not?

15. What is the smallest and biggest that their union, $P(A \cup B)$, and their intersection, $P(A \cap B)$, can be?