

1. What is the unit of observation in the dataset shown in the screengrab?
2. Sketch a plot of the distribution of the vote counts for Ahmadinejad. Label your axes and title your plot with a claim about the shape and modality of the data. Depict a shape which reflects **your** expectation of the phenomenon.
3. Sketch a plot of the distribution of the *first digit* of the vote counts for Ahmadinejad. Label your axes and title your plot with a claim about the shape and modality of the data. Depict a shape which reflects **your** expectation of the phenomenon.

Let  $X$  be a random variable which represents the first digit of vote tallies.

4. If you had no inclination as to what the probability distribution of first digits of vote counts would look like, what distribution might you assign  $X$ ?
  - State a distribution (and the values which it can take).
  - Write down the probability mass function of this distribution.

5. Sketch a probability histogram which describes the probability distribution of  $X$ . Label your axes.

6. Then calculate  $\mathbb{E}(X)$  and  $Var(X)$  for a random variable  $X$  having the distribution you assigned it in the previous question.

One common theory on how to determine whether an election is fair is as follows:

In a normally occurring, fair election, the *first digit* of the vote counts for each voting precinct should follow **Benford's Law**. If they do not, that might suggest that vote counts have been manually altered.

Benford's Law is not any universal, binding statute but actually a probability distribution on the set  $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ . Let  $Y$  be a random variable following the Benford's Law probability distribution. Then:

$$f(y) = \log_{10}\left(1 + \frac{1}{y}\right)$$

7. Sketch an (approximate) probability histogram which describes the probability distribution of  $Y$ . Label your axes.

8. Use the formula to calculate  $\mathbb{E}(Y)$ .

9. Use the formula to calculate  $Var(Y)$ .